LL methods. Both VS and LL approaches obtain the total lift by summing the forces at the FE nodal points, which was transformed from the pressure coefficients through interfaces. The VS approach transfers pressure data more accurately than the LL approach. The LL approach shows that the response around peaks deviates from the CFD solution. For this case the LL approach shows reasonable agreement with the VS approach.

The present work has strong potential for general applications dealing with more complex geometries and complete equations. Application of this approach for wing-body configurations by using the Navier–Stokes equations is demonstrated in Ref. 8.

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Shear Buckling of Simply Supported Skew Mindlin Plates

Y. Xiang*
University of Queensland, Brisbane 4072, Australia
C. M. Wang†
National University of Singapore, Kent Ridge, 0511 Singapore
and
S. Kitipornchai‡
University of Queensland, Brisbane 4072, Australia

Introduction

THERE are two kinds of in-plane shear loadings in shear buckling analysis of skew plates. The first kind of shear loading is that along the two horizontal edges where the traction is a pure shear stress, whereas along the other two oblique edges, the traction consists of both shear and direct stresses of such magnitude that every infinitesimal rectangular element is in a state of pure shear. Another kind of shear buckling considered for skew plates is where the shear loads are uniformly applied along the plate edges. Figure 1 shows these two kinds of shear loadings, and we refer to the first kind of shear loading as R shear loading and the second kind as S shear loading in this Note. The geometry of the plate is also given in Fig. 1a.

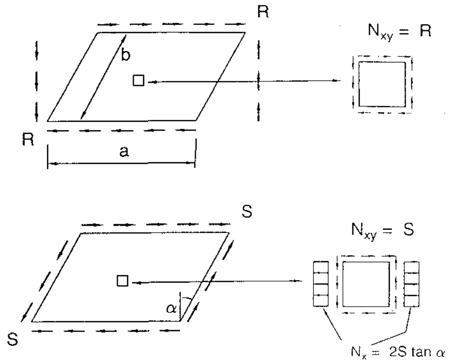

Fig. 1 Two kinds of shear loading conditions on skew plates: a) *R* shear loading and b) *S* shear loading.

Since 1954, the shear buckling of skew plates has been studied by a number of researchers such as Wittrick,[1] Argyris,[2] Durvasula,[3] and Yoshimura and Iwata.[4] These studies, however, were all confined to thin skew plates based on the Kirchhoff plate theory. When the plates are thick, these thin plate solutions overpredict the shear buckling load, and the error increases with increasing plate thickness. The error is, due to the neglect of shear deformation in thin plate theory.

Based on the Reissner–Mindlin[5,6] shear deformation plate theory and applying the pb-2 Rayleigh–Ritz solution procedure,[7,8] this Note presents critical shear load factors (for R and S shear loadings) for simply supported skew Mindlin plates of various aspect ratios a/b, skew angles α, and thickness-length ratios h/b. Note that the potential energy functional of skew Mindlin plates was derived in skew coordinates. The derivation of the formulation and the solution process have been detailed in Ref. 8. The shear correction factor $\kappa = 5/6$ and Poisson's ratio $\nu = 0.3$ are used where required.

Results and Discussion

The in-plane forces on an infinitesimal rectangular element are given by (see Fig. 1) for R shear loading

$$N_{xy} = R \text{ and } N_x = 0 \tag{1a}$$

and for S shear loading

$$N_{xy} = S \text{ and } N_x = 2S \tan\alpha \tag{1b}$$

and the shear buckling factors are for R shear loading

$$\lambda = \lambda_R = \frac{Rb^2}{\pi^2 D} \tag{2a}$$

and for S shear loading

$$\lambda = \lambda_S = \frac{Sb^2}{\pi^2 D} \tag{2b}$$

where $D = Eh^3/[12(1-\nu^2)]$ is the flexural rigidity of the plate.

Skew plates of various aspect ratios a/b, skew angles α, and thickness to oblique width ratios h/b have been considered.

Based on convergence studies, it has been found that 14th degree polynomials in the pb-2 trial functions are sufficient to ensure converged solutions. Since no shear buckling solutions for thick plates are available, comparisons can only be made with thin plate solutions. Plates with $h/b = 0.001$ may be regarded as thin plates since the shear deformation effect is almost negligible. Table 1 shows the shear buckling solutions of such thin plates with results obtained by previous researchers. Generally, the present results are in close agreement with existing solutions except for some cases (for example, $\alpha = -30$ deg). The authors believe that some of the previously published solutions may not have quite converged. The comparison studies verify somewhat the validity of the present formulation and analysis and the accuracy of the results.

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*Postdoctoral Research Fellow, Department of Civil Engineering.

†Senior Lecturer, Department of Civil Engineering.

‡Professor, Department of Civil Engineering.

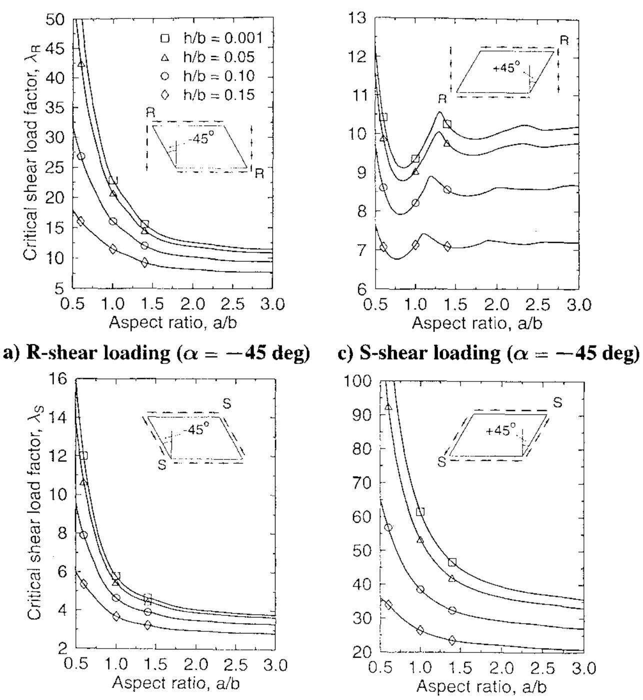
To illustrate the effects of aspect ratio, skew angle, and thickness-width ratio on the buckling loads, the variations of critical shear loads are plotted against these parameters.

Figure 2 shows variation of the critical shear load factors, λ_R and λ_S, with respect to aspect ratio for simply supported skew plates with $\alpha = -45$ and $+45$ deg. For plates under R shear loading with skew angle $\alpha = -45$ deg (Fig. 2a) and for plates under S shear loading with $\alpha = -45$ and $+45$ deg (Figs. 2c and 2d), the critical load factor decreases rapidly with respect to aspect ratio from $a/b = 0.5$ to 1.0 and thereafter decreases very gently. For plates under R shear loading with skew angle $\alpha = +45$ deg (Fig. 2b), the critical buckling load factor changes harmonically as the aspect ratio a/b increases. The plates buckle in different buckling mode shapes with different values of a/b ratios. The mode shift points are easily distinguished in this case. For the other three cases, buckling mode shift also occurs, but the mode shift points are not as obvious as the previous case. The critical load factor decreases with respect to increasing thickness-width ratio h/b for all four cases due to greater shear deformation effect. Note that the rate of decrease is more rapid when $h/b \geq 0.05$.

Table 1 Comparison case studies

α, deg	Researchers	$a/b = 1$ λ_R	$a/b = 1$ λ_S	$a/b = 2$ λ_R	$a/b = 2$ λ_S
−45	Present	22.87	5.794	12.59	4.043
	Others	27.7[a]	——	13.8[a]	——
−30	Present	16.08	5.778		
	Others	17.1[a]	6.686[b]		
−15	Present	12.12	6.835		
	Others	12.4[a]	7.063[b]		
+15	Present	7.539	14.62		
	Others	7.61[a]	14.83[b]		
+30	Present	7.150	26.87		
	Others	7.54[a] 7.07[c]	27.67[b]		
+45	Present	9.35	61.451	10.01	39.353
	Others	9.04[c] 10.7[a]	——	11.2[a]	——

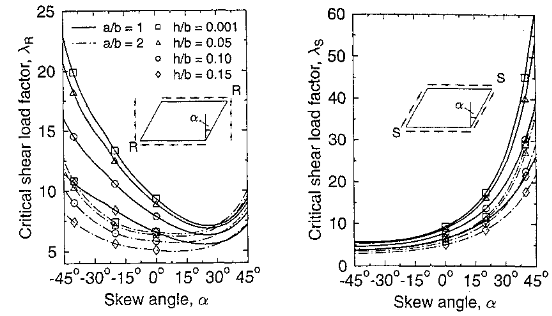
[a]Durvasula.[3]
[b]Yoshimura and Iwata.[4]
[c]Argyris.[2]



a) R-shear loading ($\alpha = -45$ deg) c) S-shear loading ($\alpha = -45$ deg)

b) R-shear loading ($\alpha = +45$ deg) d) S-shear loading ($\alpha = +45$ deg)

Fig. 2 Variation of critical shear load factors λ_R and λ_S vs the aspect ratio a/b for simply supported skew Mindlin plates under shear loadings.



a) R-shear loading b) S-shear loading

Fig. 3 Variation of critical shear load factors λ_R and λ_S vs the skew angle α for simply supported skew Mindlin plates under shear loadings.

Figure 3 shows the variations of λ_R and λ_S with respect to the skew angle α for simply supported skew plates with $a/b = 1.0$ and 2.0. It can be seen that λ_R is greater than λ_S when the skew angle $\alpha < 0$ deg, but the opposite is true when $\alpha > 0$ deg. Both λ_R and λ_S are the same when $\alpha = 0$ deg (the case of a rectangular plate). This phenomenon is explained by the fact that, from static and geometrical considerations, the R shear loading is equivalent to the S shear loading plus either 1) a tensile uniaxial load when $\alpha < 0$ deg or 2) a compressive uniaxial load when $\alpha > 0$ deg. It also follows that, for a given skew plate, the direction of the shear stresses that cause a lower critical load, λ_R, is one that tends to increase the skewness of the plate.

On the other hand, the direction of the shear stresses that cause a lower critical load, λ_S, is one that tends to decrease the skewness of the plate. Moreover, the difference between the higher and lower critical load values due to a reversal of shear stress direction becomes larger with increasing skew angles. It is thus important to take note of the direction of the shear stresses in skew plate buckling, especially when the plates are highly skewed.

Conclusions

The elastic buckling of simply supported skew Mindlin plates under shear loads has been studied, and sets of first known results have been presented. Unlike the shear buckling of rectangular plates, in skew plate buckling, 1) there are two kinds of shear loads, viz., R type and S type, and 2) the reversal of shear stress direction yields different critical shear load values. Depending on the sign of the skew angle (or the direction of the shear stresses), the critical load value λ_R may be greater or lesser than the corresponding λ_S. The relationship between these two shear loadings is explained by the fact that the R shear loading is equivalent to the S shear loading plus either a uniaxial tensile or a uniaxial compressive loading for $\alpha < 0$ and $\alpha > 0$, respectively.

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